Resonance instability of gas-dust mixture associated with dust settling in protoplanetary disc

Viacheslav Zhuravlev

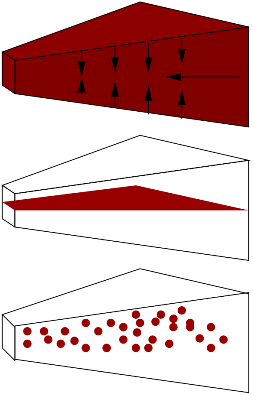
SAI MSU Moscow Russia

Dynamics and chemistry of protoplanetary discs (online), 4 March 2021

grains - aggregates - peebles - boulders - planetesimals - embryos - planets

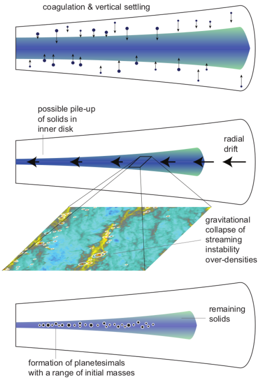
μm — sub-mm — mm — cm — m — km — 10^2 km — 10^3 km — 10^4 km

GI



Armitage (2007)

GI + SI



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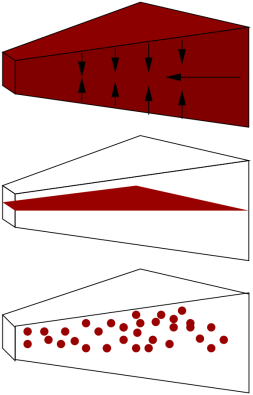
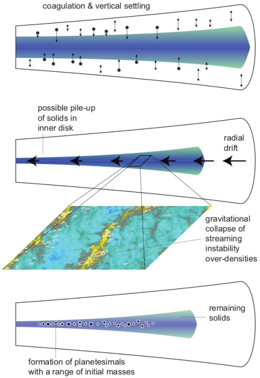


Armitage (2007)

GI + SI + ?..



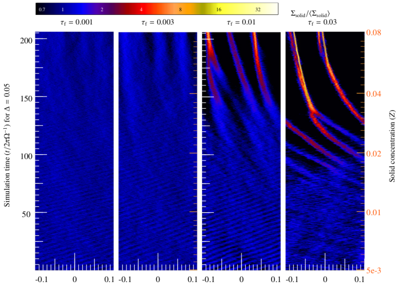
Armitage (2017)

A bottleneck for Streaming Instability

Carrera, Johansen & Davies (2015)

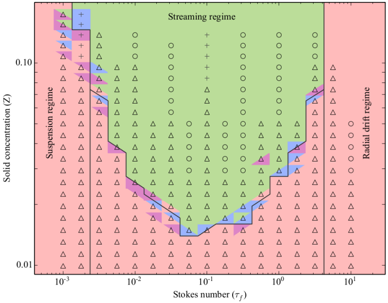
Simulations of the dust clumping due to SI
in the local laminar MMSN model

Spacetime diagram



radial coordinate (x/H)

Map of the results

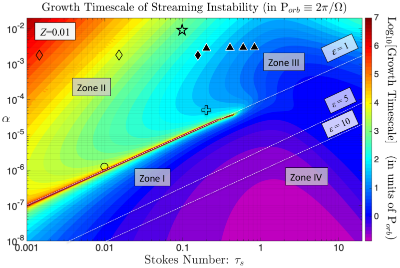


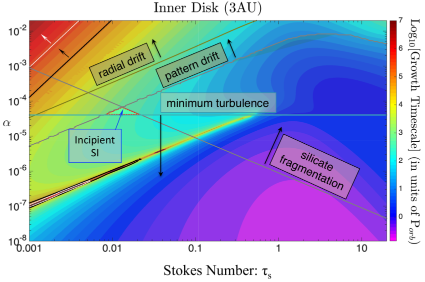
Clumping of mm-size particles barely happens
even for $Z \gtrsim 0.1$

Umurhan, Estrada & Cuzzi (2020)

Theoretical study on the relevance of SI analysing its linear growth in the local turbulent MMSN model with $\delta = 0.05$ and $Z = 0.01$

(non-stratified, settling balanced with turbulent diffusion)





Puts serious restrictions on the operation of SI

Squire & Hopkins (2018): the Streaming Instability

Small shearing box approximation: $L \ll h$.

Stationary solution for $f \equiv \rho_p/\rho_g \ll 1$:

$$\mathbf{U}_g = 0,\ \mathbf{U}_p = -t_s g_x \mathbf{e}_x,$$

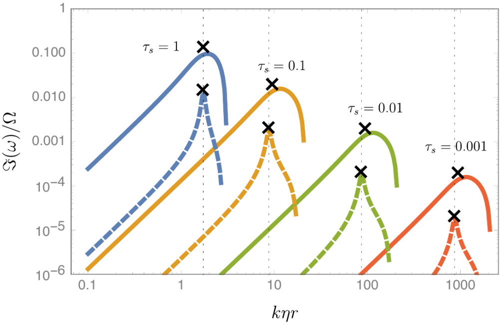
$$f = const$$

Linear axisymmetric Eulerian perturbations:

$$p',\ \rho_p',\ \mathbf{u}_g,\ \mathbf{u}_p \propto \mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}k_x x + \mathrm{i}k_z z}$$

At $\mathbf{U}_p \cdot \mathbf{k} = \pm \frac{k_z}{k}\Omega_0$:

$\Im[\omega]/\Omega_0 \propto f^{1/2}\tau$.



$\tau \equiv t_s\Omega_0$, $f = 0.1\%$ and 10%, $k^2 = k_x^2 + k_z^2$

Squire & Hopkins (2018): the Dust Settling Instability (DSI)

Small shearing box approximation: $L \ll h$.

Stationary solution for $f \equiv \rho_p/\rho_g \ll 1$:

$$\mathbf{U}_g = 0,\ \mathbf{U}_p = -t_s g_x \mathbf{e}_x - t_s g_z \mathbf{e}_z,$$

$$f = const$$

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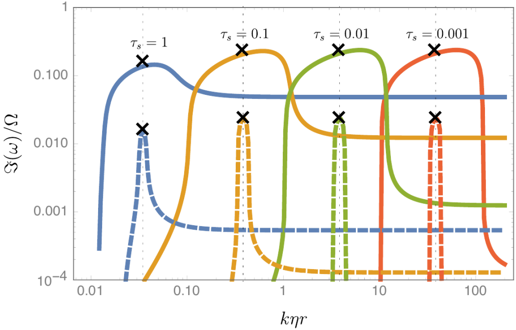

$\tau \equiv t_s\Omega_0,\quad f = 0.1\%$ and $10\%,\quad k^2 = k_x^2 + k_z^2$

Simplifying assumptions and general equations

V. Zhuravlev (2019)   doi:10.1093/mnras/stz2390

i) Small shearing box approximation $L \ll h$: $r \to x$, $\varphi \to y$, z.
ii) Axisymmetric solutions only.
iii) $\mathbf{U}_g$ and $\mathbf{U}_p$ are measured in the rotating frame with respect to $\mathbf{U}_{\rm sh} \equiv -q\Omega_0 x \mathbf{e}_y$.

$$\mathbf{U} \equiv \frac{\rho_g \mathbf{U}_g + \rho_p \mathbf{U}_p}{\rho}, \quad \mathbf{V} = \mathbf{U}_p - \mathbf{U}_g, \quad \rho \equiv \rho_g + \rho_p$$

$$\tau \equiv t_s \Omega_0 \ll 1$$

$$h \gg l_{\rm pert} \gg l_s \equiv g t_s^2$$

Terminal velocity approximation

$$\partial_t \mathbf{U} - 2\Omega_0 U_y \mathbf{e}_x + (2-q)\Omega_0 U_x \mathbf{e}_y + (\mathbf{U}\cdot\nabla)\mathbf{U} = \mathbf{g} - \frac{\nabla p}{\rho},$$

$$\frac{\nabla p}{\rho} = \frac{\mathbf{V}}{t_s}$$

$$\nabla \cdot \mathbf{U}_g = 0,$$

$$\partial_t \rho_p + \nabla \cdot (\rho \mathbf{U}) = 0.$$

Stationary solution and equations for perturbations

$$\mathbf{U} = 0$$

$$\frac{\nabla p}{\rho} = -g_x \mathbf{e}_x - g_z \mathbf{e}_z$$

$$\mathbf{V} = -t_s (g_x \mathbf{e}_x + g_z \mathbf{e}_z)$$

$$f \equiv \rho_p / \rho_g = const \ll 1$$

Perturbation variables: $\varpi \equiv -\partial_z u_y$ $\phi \equiv \partial_z u_x$ u_z $\delta \equiv \rho_p' / \rho_p$

$$\partial_t \phi = \partial_{tx}^2 u_z - 2\Omega_0 \varpi + f(g_z \partial_x \delta - g_x \partial_z \delta)$$

$$\partial_t \varpi = \frac{\kappa^2}{2\Omega_0} \phi$$

$$\partial_{tx}^2 \varpi = -\frac{\kappa^2}{2\Omega_0} \partial_{zz}^2 u_z$$

$$\partial_{tz}^2 \delta = t_s (g_x \partial_{xz}^2 \delta + g_z \partial_{zz}^2 \delta) + 2\tau \partial_x \varpi$$

$$\kappa^2 = 2(2-q)\Omega_0^2$$

Consider modes of gas-dust perturbations $\chi_j = \hat{\chi}_j e^{-\mathrm{i}\omega t + \mathrm{i}k_x x + \mathrm{i}k_z z}$

The variational principle for neutral modes of perturbations (Whitham 2011):

$$\delta \int L(\hat{\chi}_j, \partial_{t,x,z}\theta)\, dx\, dz\, dt = 0, \quad \text{where } \theta \equiv -\omega t + k_x x + k_z z,$$

shows that an algebraic equations for $\hat{\chi}_j$ are equivalent to

$$\text{the Euler-Lagrange equations:} \quad \frac{\partial L}{\partial \hat{\chi}_j} = 0.$$

This yields an explicit form of the Largangian

$$L = L_0 + f\, L_1 = 0,$$

$$L_0 = \omega \hat{\varpi} \hat{\phi} + \omega k_x \hat{\varpi} \hat{u}_z - \Omega_0 \hat{\varpi}^2 - \frac{\kappa^2}{2\Omega_0} \frac{\hat{\phi}^2}{2} - \frac{\kappa^2}{2\Omega_0} \frac{k_z^2 \hat{u}_z^2}{2},$$

$$L_1 = g_z \left[1 - \frac{k_z}{k_x} \frac{g_x}{g_z} \right] \left\{ k_x \hat{\varpi} \hat{\delta} + \frac{k_z \hat{\delta}^2}{4\Omega_0} \left[\frac{\omega}{t_s} + (g_z k_z + g_x k_x) \right] \right\}.$$

The fundamental symmetry of L with respect to translations in time yields the mode energy:

$$E \equiv \omega \frac{\partial L}{\partial \omega}$$

This yields an explicit form of the energy

$$E = E_0 + f\,E_1,$$

$$E_0 = 2\Omega_0 \frac{k^2\omega^2}{\kappa^2 k_z^2}\,\hat{\varpi}^2,$$

$$E_1 = \frac{\omega}{4\tau}\frac{k_z}{k_x}(k_x g_z - k_z g_x)\,\hat{\delta}^2.$$

As soon as $f > 0$ and $\hat{\delta} > 0$, the mode energy is NOT positive definite.

$$D_g(\omega, \mathbf{k}) \cdot D_p(\omega, \mathbf{k}) = \epsilon(\mathbf{k}),$$

Inertial Waves (IW): $$D_g(\omega, \mathbf{k}) \equiv \omega^2 - \omega_i^2 = 0,$$

Streaming Dust Wave (SDW): $$D_p(\omega, \mathbf{k}) \equiv \omega - \omega_p = 0,$$

$$\omega_i \equiv \pm \frac{k_z}{k} \kappa,$$

$$\omega_p \equiv -t_s (k_x g_x + k_z g_z),$$

The coupling term: $$\epsilon(\mathbf{k}) \equiv f t_s \kappa^2 \frac{k_x}{k} \frac{k_z}{k} (k_x g_z - k_z g_x).$$

The mode crossing (resonance condition):

$$\omega_p = \omega_i \equiv \omega_c$$

An estimate of the growth rate (cf. Squire & Hopkins 2018):

$$\mathrm{Im}[\omega] \approx \pm \left(\frac{\epsilon}{\partial_\omega D_g|_{\omega_c} \cdot \partial_\omega D_p|_{\omega_c}} \right)^{1/2} = \pm \left(\frac{\epsilon}{2\omega_c} \right)^{1/2} = O\left(f^{1/2}\right)$$

The instability arises if and only if energy of SDW is negative

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As $g_z \to 0$, energy of SDW becomes positive definite.

RDI disappears within TVA!

The mode crossing (resonance condition):

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The instability arises if and only if energy of SDW is negative

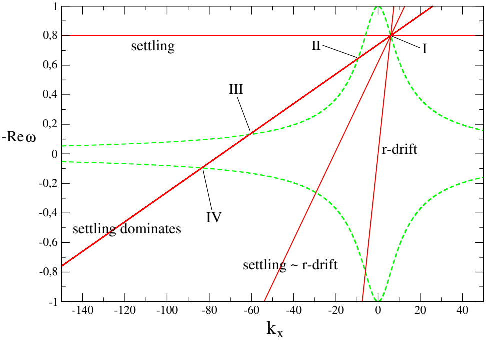
As $g_z \to 0$, energy of SDW becomes positive definite.

RDI disappears within TVA!

SI of Youding & Goodman (2005) is recovered retaining in $\epsilon(\mathbf{k})$ the higher-order terms over $\tau \ll 1$. That is why $\mathrm{Im}[\omega_{\mathrm{SI}}] = O(f^{1/2}\tau)$.

$f = 0$

The red curve: SDW
The green curve: IW



Everywhere below: $q = 3/2$, $\tau = 0.1$ and $k_z > 0$.
Units: Ω_0^{-1}, an artificial $L < h$.

The dust settling only (type-I crossing)

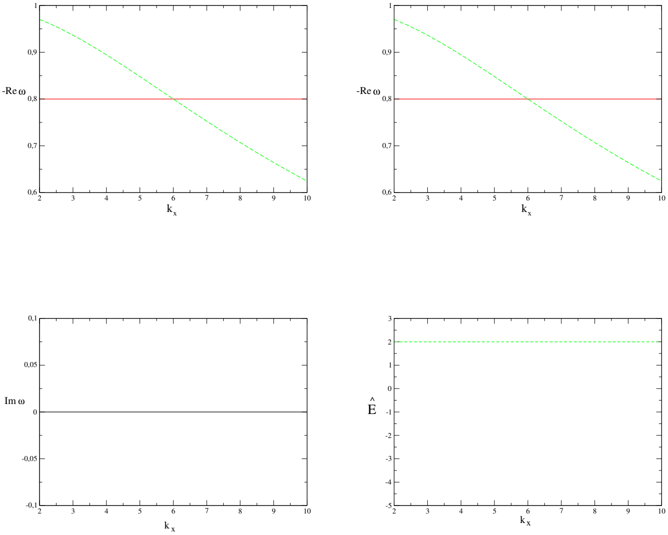
$f = 0.0$

$g_x = 0$

$g_z = 1.0$

$k_z = 8$

Mode coupling getting stronger



The dust settling only (type-I crossing)

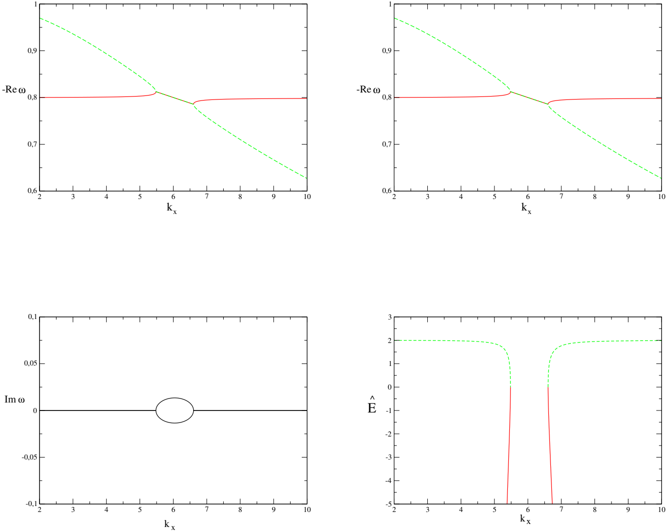
$f = 0.001$

$g_x = 0$

$g_z = 1.0$

$k_z = 8$

Mode coupling getting stronger



The dust settling only (type-I crossing)

$f = 0.01$

$g_x = 0$

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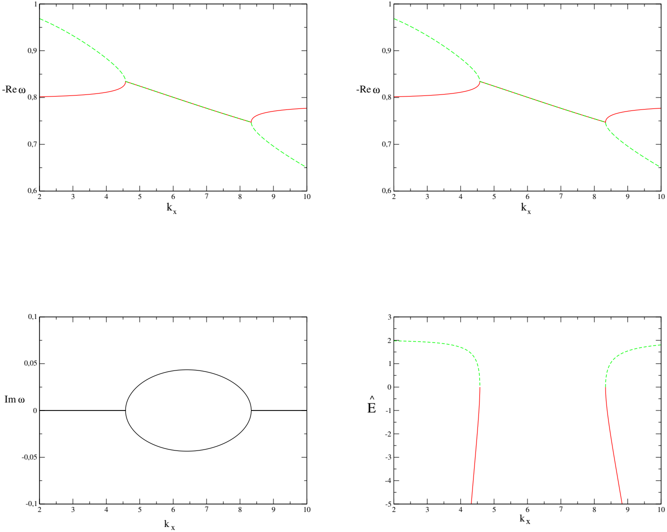
Mode coupling getting stronger



Adding the radial drift (type-I crossing)

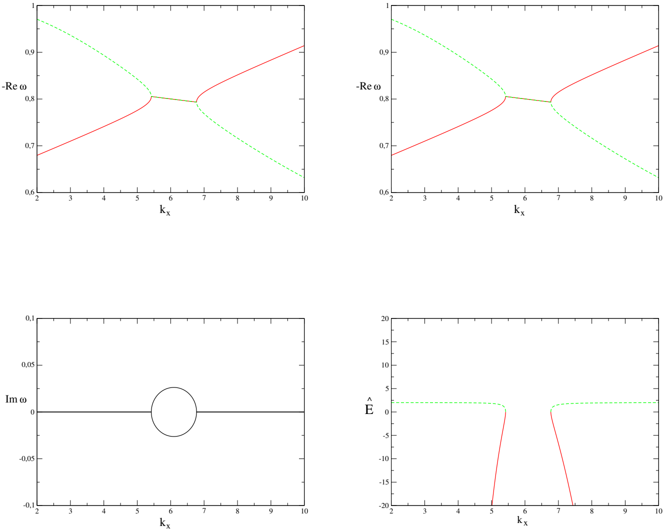
$f = 0.01$

$g_x = 0.3$

$g_z = 0.775$

$k_z = 8$

Mode coupling $\rightarrow$
Avoided crossing



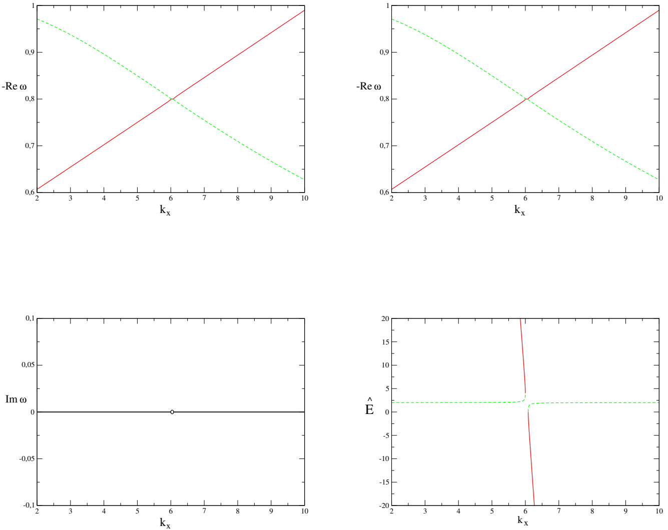
$f = 0.01$

$g_x = 0.48$

$g_z = 0.64$

$k_z = 8$

Mode coupling $\rightarrow$
Avoided crossing



Adding the radial drift (type-I crossing)

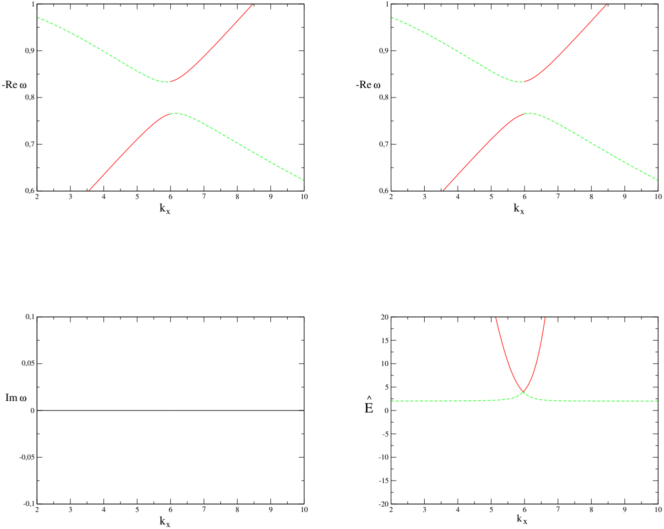
$f = 0.01$

$g_x = 0.8$

$g_z = 0.4$

$k_z = 8$

Mode coupling $\rightarrow$
Avoided crossing



The dispersion equation in TVA

V. Zhuravlev (2020) doi:10.1093/mnras/staa805

$$D_g(\omega', \mathbf{k}) \cdot D_p(\omega', \mathbf{k}) = \epsilon(\mathbf{k}),$$

Inertial Waves (IW):

Streaming Dust Wave (SDW):

$$D_g(\omega', \mathbf{k}) \equiv (\omega' + \mathrm{i}\omega_*)^2 - \omega_i^2 = 0,$$

$$D_p(\omega', \mathbf{k}) \equiv \omega' - \omega_p = 0,$$

$$\omega_* \equiv \omega_\nu - \omega_D, \quad \text{where} \quad \omega_{\nu,D} \equiv \{\nu, D\} k^2.$$

$$\omega \equiv \omega' - \mathrm{i}\omega_D.$$

In the particular case of $Sc \equiv \nu / D = 1$: $\Im[\omega] = -\mathrm{i}\omega_\nu + \Delta$,

where Δ is the inviscid estimate of the growth rate.

Dust Settling Instability in a turbulent disc: $\nu = D$

Settling

The mode crossing
in the long wavelength limit:

$$k_z \ll \tilde{k}_z$$

$$k_x \approx \tilde{k}_z$$

$$\alpha_{max} \approx \left(\frac{f}{2}\right)^{1/2} \left(\frac{z_0}{h}\right)^2 \tau^2$$

Settling + (small) Radial drift

The mode crossing
in the long wavelength limit:

$$k_z \ll k_x \ll \tilde{k}_x$$

$$k_x \approx (k_z \tilde{k}_x)^{1/2}$$

$$\alpha_{max} \approx f^{1/2} \left(\frac{z_0}{h}\right)^{5/4} \tau^{5/4} \left[\frac{\eta}{(k_z z_0)\delta_*}\right]^{3/4}$$

Definitions

$$\tilde{k}_{x,z} \equiv \frac{\kappa}{t_s\, g_{x,z}} \sim (\tau z_0)^{-1} \quad \text{and} \quad \nu \equiv \alpha \Omega_0 h^2$$

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$$k_x \approx \tilde{k}_z$$

$$\alpha_{max} \approx \left(\frac{f}{2}\right)^{1/2} \left(\frac{z_0}{h}\right)^2 \tau^2$$

Turbulence prevents settling:

$$\alpha_{stl} \simeq \tau \left(\frac{z_0}{h}\right)^2$$

Settling + (small) Radial drift

The mode crossing in the long wavelength limit:

$$k_z \ll k_x \ll \tilde{k}_x$$

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Definitions

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Viscous Settling Instability

As $\nu > D$,
SDW and IW
decouple from each other.

SDW saves its growth
at the mode crossing
even as $\nu - D \to \infty$.

SDW becomes growing
outside the band of DSI

$\omega \equiv \omega' - i\omega_D$.

$f = 0.01$

$g_x = 0$

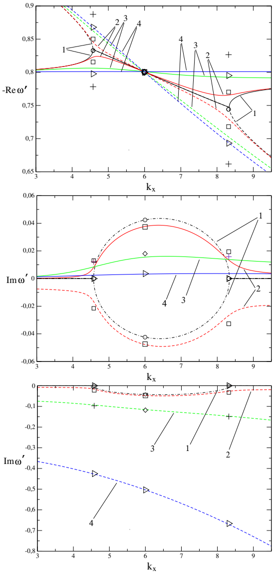
$g_z = 1.0$

$k_z = 8$

The curves

black: $\nu - D = 0$,
red: $\nu - D = 0.0001$,
green: $\nu - D = 0.001$
blue: $\nu - D = 0.005$

SDW: solid line
IW: dashed line
The coupled modes:
dot-dashed line



Viscous Settling Instability

As $\nu > D$,
SDW and IW
decouple from each other.

SDW saves its growth
at the mode crossing
even as $\nu - D \to \infty$.

SDW becomes growing
outside the band of DSI
including the long
wavelengths.

$\omega \equiv \omega' - i\omega_D$.

$f = 0.01$

$g_x = 0$

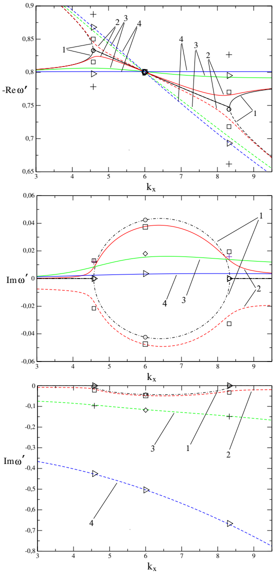
$g_z = 1.0$

$k_z = 8$

The curves

black: $\nu - D = 0$,
red: $\nu - D = 0.0001$,
green: $\nu - D = 0.001$
blue: $\nu - D = 0.005$

SDW: solid line
IW: dashed line
The coupled modes:
dot-dashed line



The long wavelength limit $k \ll k_c = \tilde{k}_z$.

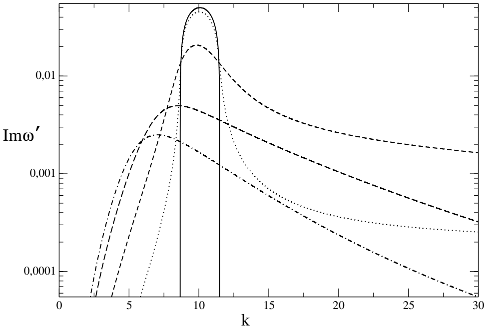
In the case of small viscosity,

$$\hat{\omega}_* = \alpha(1 - Sc^{-1}) \frac{k}{k_z} \frac{k^2}{\tilde{k}_z^2} \left(\frac{z_0}{h}\tau\right)^{-2} \lesssim 1,$$

$$\Im[\omega] \approx \frac{\omega_\nu}{Sc} \left[2f \frac{k_x^2}{k_z^2} \frac{k^2}{\tilde{k}_z^2}(Sc - 1) - 1\right]$$

$\Im[\omega] > 0$ until

$$\alpha_{max} \approx \frac{Sc}{(Sc-1)^{1/4}} \frac{f^{3/4}\tau^{3/2}}{(k_z z_0)^{1/2}} \left(\frac{z_0}{h}\right)^2$$



Exact solution of the dispersion equation.

$f = 0.01,\ g_x = 0,\ g_z = 1.0,\ \theta = 45^o.$

The long wavelength limit $k \ll k_c = \tilde{k}_z$.

In the case of small viscosity,

$$\hat{\omega}_* = \alpha(1 - Sc^{-1}) \frac{k}{k_z} \frac{k^2}{\tilde{k}_z^2} \left(\frac{z_0}{h} \tau \right)^{-2} \lesssim 1,$$

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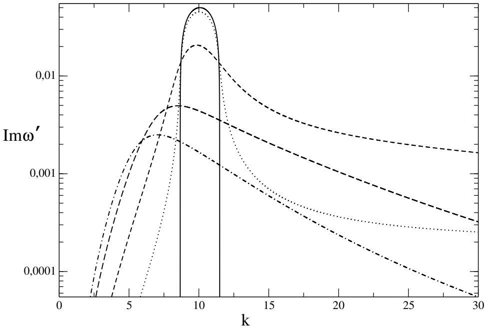
$\Im[\omega] > 0$ until

$$\alpha_{max} \approx \frac{Sc}{(Sc-1)^{1/4}} \frac{f^{3/4} \tau^{3/2}}{(k_z z_0)^{1/2}} \left(\frac{z_0}{h} \right)^2$$

Compare with the inviscid

$$\alpha_{max} \approx \left(\frac{f}{2} \right)^{1/2} \left(\frac{z_0}{h} \right)^2 \tau^2$$



Exact solution of the dispersion equation.

$f = 0.01$, $g_x = 0$, $g_z = 1.0$, $\theta = 45^o$.

- The dust settling in protoplanetary disc gives birth to modes of gas-dust perturbations having negative energy. The basic case is represented by the advection of the dust density perturbations, which is called SDW.
- This allows for the mode coupling between SDW and IW, which leads to DSI on the local scale smaller than the disc scaleheight. To the main order in the small dust fraction, DSI is RDI in TVA. In the absence of the dust settling there is no RDI in TVA.
- SI of Youding & Goodman (2005) arises with the account of terms beyond TVA, i.e. due to inertia of solids.
- DSI in a turbulent disc is suppressed at higher threshold viscosity as compared to SI, especially with an account for combination of the dust settling and the dust radial drift.
- As turbulent viscosity exceeds turbulent diffusion, SDW decouples from IW and becomes growing outside the band of DSI giving birth to VSI on the side of long waves.

Thank you!